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A Framework for Robust Run by Run Control with Lot Delayed Measurements

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A Framework for Robust Run by Run Control with Lot Delayed Measurements

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Abstract—This paper considers the run by run control problem. We develop a framework to solve such a problem in a robust fashion. The framework also encompasses the case when the system is subject to delayed measurements. Recent results available for the control of such systems are reviewed, and two examples are presented. The first example is based on the end-pointing problem for a deposition process, and is subject to noise which has both Gaussian and uniform components. The second one is concerned with rate control in an LPCVD reactor.

I. INTRODUCTION

RECENTLY, there has been a strong interest in RbR control in the semiconductor industry. With device tolerances shrinking, it becomes necessary to squeeze maximum performance out of existing equipment. A further advantage of the RbR control framework is that it enables automatic recipe generation to meet different targets, and also aids in the recovery of the process after a large disturbance. Furthermore, RbR control could also prove useful in extending the time between maintenance shut downs (for example, one may be forced to periodically clean the equipment due to process drift induced by deposit build up in the chamber). In this paper, we present a worst case framework for carrying out RbR control. The advantage of this approach is its ability to handle uncertainty. This is useful in cases when we do not have confidence in our models, such as after a sudden change in process characteristics. Furthermore, the set theoretic approach followed allows us to relax assumptions on the statistics of the noise. This becomes important when one is working with small sample sizes, and can no longer appeal to the central limit theorem. Moreover, the current approach also yields guaranteed performance bounds and allows one to deal with nonlinear models.

In addition to the above, we will also consider the case when we have multiple delays between lots (runs). This case is of importance in high volume processing lines. Here one wants to maximize equipment utilization, and this implies initiating the next lot before measurements can be carried out on the current lot.

As with any control strategy, some a priori information needs to be available about the process model. What we require is the structure of the map between the recipe and the measured variables. Such maps could be provided by models obtained via the response surface methodology (RSM) [2]. A number of researchers have successfully employed RSM to the problem of automated recipe generation, process optimization, and design [3]–[9]. However, the conceptual development is not restricted to such models alone.

In addition to the above, we will also need bounds on the process noise, in the sense that the assumed model, with these noise bounds, can account for $(1 - \alpha)\%$ of the observations, where $\alpha$ is a very small number. For example, if we choose $\alpha$ to be 0.27, then the model with these noise bounds can account for 99.73% of the observations. The problem of selecting these bounds is similar to the problem of specifying control limits for control charts in statistical process control (SPC).

The idea here is, that if we define our process model in this manner, then the RbR controller hardly ever observes process results which are inconsistent with the model. The influence of these bounds on the performance of the RbR controller is similar to that observed in control charts. For example, if the bounds are chosen to be smaller than what they actually are, the controller will generate an alarm via a consistency check (see Section II-C), even if the process is in control. On the other hand if they are too lax, then the controller becomes less sensitive to process variation. The idea behind this approach is illustrated in Fig. 1. For the sake of clarity, we consider a single-input single—output system. Fig. 1 plots the model prediction $\hat{y}(x, u)$ against the recipe settings $(u)$. The dots represent the actual measurements $(y)$, which are scattered around the predicted output. Typically, one tries to account for this scatter by assuming that the system is subject to random noise. However, we will represent the noise as deterministic but bounded, with the noise bounds being given by the dashed lines in Fig. 1. Of course, by making such an assumption, the possibility of outliers (points which violate the assumed bounds) exists, and one such outlier is shown in

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Fig. 1. Predicted output (solid), actual measurements (dots), and noise bounds (dashed).

This reflects the scenario where the statistics of variation in the quality of a particular raw material vary with different suppliers, but this variation in quality is forced to satisfy bounds imposed by quality constraints (i.e., certain statistics, but fixed known bounds).

The rest of the paper is organized as follows. In Section II, we state the problem and review the relevant theory that is employed to solve it. The conceptual solution to the problem is obtained via dynamic programming [10]. Two examples are presented in Section III, which illustrates how the RbR control problem could be recast in the given framework, and some simulation results are presented. The first example concerns the problem of end-pointing and is developed in detail. The second one is concerned with the problem of rate control in an LPCVD reactor.

II. THE FRAMEWORK

We can set up the RbR control problem as the following dynamical system:

\[
x_{k+1} \in \mathcal{F}(x_k, u_k), \quad x_0 \in X_0
\]
\[
y_{k+1} \in \mathcal{G}(x_{k-\tau}, u_{k-\tau})
\]
\[
z_{k+1} = l(x_{k+1}, u_k).
\]

This formulation is motivated by the discussion in the Introduction. We will see in the next section via examples how the problem can be set up in this fashion. Here, \(x_k \in R^n\) are the states, \(u_k \in U \subseteq R^m\) are the vectors of recipes, \(y_k \in R^f\) are the measurements, and \(z_k \in R^g\) are the regulated outputs. Furthermore, \(X_0 \subseteq R^n\) represents the set of possible initial states. Note that the measurement is subject to a finite delay \(\tau \geq 0\), where \(\tau = 0\) represents the delay-free case. The set \(U\) of allowable recipes can be determined from various considerations. One criteria clearly is the set of recipes where the experimentally determined model is valid. Certain assumptions have to be made on the system (1) [11], [12]. In particular, we require that the sets \(\mathcal{F}(x, u), \mathcal{G}(x, u), \text{and } U\) be compact (i.e., closed and bounded), and that \(l(x, u)\) be continuously differentiable in \(x\), for all \(u \in U\). Also, we require that the set

\[
\mathcal{L}^\gamma \triangleq \{ u \in R^f; \left| \frac{\partial}{\partial x} l(s, u) \right| \leq \gamma, \text{ for some } u \in U \}
\]

be compact, and contains the origin for all \(\gamma > 0\). Here, \(\left| \frac{\partial}{\partial x} l(s, u) \right| \) denotes the Euclidean norm. This assumption precludes certain kinds of cost functions, e.g., linear. However, a common class which does satisfy this assumption are functions quadratic in the states. These are what we employ in the examples. Furthermore, we require that \(\mathcal{F}(x, u)\) is never a singleton (i.e., single-valued), and that zero is an equilibrium point for the system in the sense that

\[
0 \in \mathcal{F}(0, 0);
\]
\[
0 \in \mathcal{G}(0, 0);
\]
\[
l(0, 0) = 0.
\]

During run \(k\), one is concerned with selecting a recipe \(u_k\), so as to regulate \(z_{k+1}\) in a suitable fashion. A framework for dealing with such systems has been established in [11] and [12]. The aim of the controller is to guarantee the following inequality for all trajectories \(s\) and \(r\) that can be generated by system (1):

\[
\sum_{i=0}^{\infty} |l(s_{i+1}, u_i) - l(r_{i+1}, u_i)|^2 - \gamma^2 |s_{i+1} - r_{i+1}|^2 \leq \beta^u(x_0)
\]

\[
(2)
\]

where \(\gamma > 0\) is specified \textit{a priori}, and \(\beta^u(x)\) is a finite quantity with \(\beta^u(x) \geq 0\), and \(\beta^u(0) = 0\). This objective has the following interpretation [11]. Let \(r - s \in l^2(0, \infty), R^n\), i.e., \(\|r - s\| = (\sum_{i=0}^{\infty} |r_i - s_i|^2)^{1/2} < \infty\). Then, if \(x_0 = 0\), satisfaction of (2) implies that

\[
\|l(s, u) - l(r, u)\| \leq \gamma \|r - s\|
\]

which illustrates that one is trying to minimize the variation of the regulated output with respect to the variation in the state trajectories. The latter is caused by the various noise sources affecting the system (this is made clearer via examples in Section III). Furthermore, (2) also implies that for any \(x_0 \in X_0\), one has

\[
x_k \to \mathcal{L}^\gamma \text{ as } k \to \infty
\]

and hence the system is ultimately bounded.

For the case when we have perfect state information (i.e., we know the value of \(x_k\) for every \(k\)) we can solve for the controller by solving the following stationary dynamic programming equation [11], [12]:

\[
V(x) = \inf_{u \in U} \sup_{r \in \mathcal{F}(x, u)} \{ |l(r, u) - l(s, u)|^2 \}
\]

\[
- \gamma^2 |r - s|^2 + V(r)
\]

\[
(3)
\]
to obtain \( V(x) \geq 0 \), with \( V(0) = 0 \). However, in general we never have perfect state information and must consider the measurement feedback case. We first consider the case when we have no delay (i.e., \( \tau = 0 \)).

### A. The Delay-Free Case

We define, for an observed measurement trajectory \( y_{1:k} \) and recipe trajectory \( u_{0:k-1} \), an information state \( p_k \), as follows [11]

\[
p_k(x) = \sup_{x_0 \in X_0} \sup_{r \in F_0^j(x_0)} \left\{ p_0(x_0) + \sum_{i=1}^{k} \left| l(r_i, u_{i-1}) \right| - l(r_i, u_{i-1})^2 \right\}
\]

where, \( F_0^j(x_0) \) is the set of state trajectories generated by the system (1), compatible with the observed \( y_{1:k} \) and \( u_{0:k-1} \), given that the initial state was \( x_0 \). See [11] and [12] for more details. Here, the convention is the that the supreme over an empty set is \(-\infty\). \( p_0 \) defines a weighing on the initial value of the states \( x_0 \) and could include any a priori information available about the initial conditions. Since \( x_0 \in X_0 \), we may assume that \( p_0(x) = -\infty \), for all \( x \notin X_0 \). We now define

\[
H(p, u, y)(x) = \sup_{\xi \in \mathbb{R}^n} \{ p(\xi) + B(\xi, x, u, y) \}
\]

with the function \( B \) defined by

\[
D(\xi, x, u, y) \triangleq \sup_{s \in \mathcal{F}(\xi, v)} \{ |l(x, v) - l(s, v)|^2 - \gamma^2|x - s|^2 \}
\]

if \( x \in \mathcal{F}(\xi, v) \), \( y \in \mathcal{G}(\xi, v) \), and is equal to \(-\infty\) elsewhere. It can then be shown ([11], [12]) that we can recursively compute \( p_k \) as

\[
p_{k+1} = H(p_k, u_k, y_{k+1}), \quad k = 0, 1, \ldots
\]  

(4)

for some initial \( p_0 \). Here, it should be noted that \( p_k \) is actually a function of \( x \), and hence the dynamic system (4) is infinite dimensional in general. We can then, in principle, obtain the measurement feedback controller via the following stationary dynamic programming equation [11]

\[
M(p) = \inf_{u \in U} \sup_{y \in \mathbb{R}^n} M[H(p, u, y)].
\]

(5)

In practice however, solving (5) is computationally hard, since the equation is inherently infinite dimensional. Hence, we implement a certainty equivalence controller [13]–[15]. At time \( k \) we have \( p_k \), \( V \) the solution to (3), and the corresponding state feedback policy \( u_F \). We then compute

\[
\hat{x}_k = \arg \max_{z \in \mathbb{R}^n} \{ p_k(z) + V(z) \}
\]

(6)

and implement \( u_k = u_F(\hat{x}_k) \). Note that in general the certainty equivalence controller is nonoptimal. However, some conditions are available which enables one to check for optimality [14], [15]. Furthermore, in [15], a family of controllers is developed which enables one to meet the performance requirements when the optimality conditions for certainty equivalence fail.

The controller obtained depends on \( \gamma \). In particular, one would want to iteratively test different values of \( \gamma \) and choose the smallest value such that (5) has a solution. Let \( M(p) \geq \sup_{x \in \mathbb{R}^n} p(x) \), for every \( p \), and \( M(p_0) = 0 \). However, doing so is nontrivial. On the other hand note that the value of \( \gamma \) is lower bounded by \( \gamma^* \), where \( \gamma^* \) is the smallest value of \( \gamma \) such that (3) has a solution \( V \), with \( V(x) \geq 0 \). This is due to the fact that the performance for the state feedback case is no worse than the performance for the measurement feedback case where one does not have perfect state information. Hence, the value of \( \gamma \) should at least be \( \gamma^* \).

### B. Delayed Measurements

So far, we assumed that sufficient time is available between lots to carry out measurements and to calculate the new recipe. In general, however, we may have a delay of multiple lots. Let this be denoted by \( \tau \geq 0 \), where \( \tau \) is the number of lots after which the measurement of the current lot becomes available. Note that the delay-free case is a special instance of the general delayed measurement problem. A detailed framework for dealing with such systems can be found in [16], where necessary and sufficient conditions for the solvability of the problem are stated. Moreover, the general idea is similar to [17].

The objective of the controller design still remains as in (2). However, the certainty equivalence controller is modified as follows. We first compute

\[
\hat{x}_k \in \arg \max_{z \in \mathbb{R}^n} \{ J(\hat{p}_k(z)) + V(z) \}
\]

and then implement \( u_k = u_F(\hat{x}_k) \), where \( V \) is again the solution to (3) and \( u_F \) is the corresponding state feedback policy. Here

\[
\hat{p}_k(z) = \begin{cases} p_0(z) & \text{if } k \leq \tau \\ p_{k-\tau}(z) & \text{else} \end{cases}
\]

where \( p_k \) is recursively computed via a delayed version of (4) as

\[
p_{k-\tau+1} = H(\hat{p}_{k-\tau}, u_{k-\tau}, y_{k+1}), \quad k = \tau, \tau + 1, \ldots
\]

with \( H \) and \( B \) defined as before. Also \( J(\hat{p}_k) \) is defined as

\[
J(\hat{p}_k(z)) = \sup_{z_0 \in \mathbb{R}^n} \{ \hat{p}_k(z_0) + Q_0(z_0, x) \}
\]
where $Q_0(x_0, z)$ is the solution of the following dynamic programming equation:

$$Q_1(\xi, x) = \sup_{r, u \in \mathcal{F}(\xi, u)} \{l(r, u) - l(s, u)\}^2 - \gamma^2 r - s^2 + Q_{i+1}(r, x)
- (k - \min\{r, k\}) \leq i \leq k - 1$$

and we set $Q_0(x_0, z) = Q(\xi, L(k, k)) (x_0, z).

C. A Special Case

So far we have assumed that the system conforms to the model assumptions. One would, however, like a course of action in case the system does in fact violate the model assumptions. If the model has been correctly defined, the probability of this occurring should be negligible (i.e., $\ll 1\%$), and we may never observe a violation in practice. However, if the model is incorrectly defined, the chances of violation increase.

We can check for violation in the following manner. Let the measurement delay be $\tau$. Assume we have just finished with run $k$. Then the controller flags a violation if given the current information state $p_{k-1}$, the recipe $u_{k-1}$, and the measurement $y_{k+1}$

$$H(p_{k-1}, u_{k-1}, y_{k+1}) = -\infty, \quad \text{for all } x \in \mathbb{R}^n.$$

This test and the subsequent re-initialization of the information state, is based on the fact that the information state is a weighed indicator function of the set of feasible states [18], and takes the value $-\infty$ for all infeasible states. To recover from this situation, we reinitialize the information state. At this time, all we can say is that the process has either shifted by an exceptional amount, or that $y_{k+1}$ is an exceptionally bad data point. Note that we are using the term exceptional to classify this situation, since the model already incorporates both expected measurement errors and process shifts. To reinitialize the information state, we do the following:

1) Define $\hat{p}_{k-1}$ in the following manner

$$\hat{p}_{k-1}(x) \triangleq \begin{cases} p_{k-1}(x) & \text{if } p_{k-1}(x) \neq -\infty \\ 0 & \text{if } p_{k-1}(x) = -\infty \\ \text{and } y_{k+1} \in \mathcal{V}(x, u_{k-1}). \end{cases}$$

The first case deals with the possibility that $y_{k+1}$ is a bad data point, whereas the latter deals with the possibility of an exceptional shift.

2) Now define $p_{k+1}$ as

$$p_{k+1}(x) \triangleq \sup_{\xi \in \mathbb{R}^n} \{\hat{p}_{k-1}(\xi) + \hat{B}(\xi, x, u_{k-1})\}$$

where $\hat{B}$ is defined as

$$\hat{B}(\xi, x, u) \triangleq \sup_{s \in \mathcal{F}(x, u)} \{|l(x, u) - l(s, u)|^2 - \gamma^2 |x - s|^2\}$$

If $x \in \mathcal{F}(\xi, u)$, or else is equal to $-\infty$. $\hat{B}$ defined in this fashion deals with the possibility that an exceptional shift has occurred.

The information state is now propagated as before from the next run onwards, unless another violation is flagged.

III. EXAMPLES

In this section, two examples are presented. The first one deals with end-pointing a deposition process, and is developed in detail. The second example deals with the problem of rate control in an LPCVD reactor, where the controller has to deal with multiple objectives.

A. End-Pointing

In this subsection, we consider a simple example, i.e., the problem of end-pointing. The scenario is as follows. Lots, consisting of 24 wafers are processed through a single wafer reactor. Here, we assume that the process under consideration is deposition. Measurements are carried out on the last wafer of each lot. The aim is to determine the processing time, so as to achieve a given target thickness. Here, it is assumed that the processing time per wafer is constant for all wafers across a lot. We assume that the process is subject to three kinds of noise: 1) variation in the average deposition rate at the test wafer from lot to lot; 2) variation in the instantaneous rate from test wafer to test wafer, due to changes in both the wafer surface, and deposition conditions, and 3) measurement noise, either due to finite resolution of the measurement apparatus, or due to experimental error. Here, the basic process can be modeled as (assuming for now that we have no measurement delay)

$$\hat{r}_{k+1} = \hat{r}_k + v_k$$
$$\hat{z}_{k+1} = (\hat{r}_k + w_k) \hat{t}_k$$
$$\hat{y}_{k+1} = (\hat{r}_k + w_k) \hat{t}_k + m_k$$
$$\hat{T}_{k+1} = \hat{T}_k$$

where $\hat{r}_k$ is the average deposition rate for the test wafer in lot $k$, $\hat{z}_{k+1}$ is the actual deposition thickness on the test wafer for lot $k$ for a deposition time $\hat{t}_k$, and $\hat{y}_{k+1}$ is the measured thickness. Also, $\hat{T}_k$ is the target thickness for lot $k$. Here, $v_k$ is the noise used to model the variation in the average deposition rate, $w_k$ is the noise used to model the rate variation per wafer, and $m_k$ is the noise modeling the measurement error. It is assumed that the controller knows $\hat{T}_k$ before the processing time for lot $k$ is computed.

We now give some (fictitious) numbers to enable simulations. It is assumed that the nominal (or average) thickness required is $\bar{T} - \xi = 1500$ A. Furthermore, the nominal deposition rate of the system is assumed to be $\bar{r} = 300$ A/min, and hence the nominal deposition time is $\bar{t} = 5$ min. Hence, we can now express the system in terms of deviations from the nominal, i.e., as

$$r_{k+1} = r_k + v_k$$
$$z_{k+1} = r_k t_k + \bar{r} t_k + (\bar{t} + t_k) w_k$$
$$R_{k+1} = T_k$$
$$y_{k+1} = r_k t_k + \bar{r} t_k + (\bar{t} + t_k) w_k + m_k$$

where $r_k = \bar{r} - \bar{r}, t_k = \bar{t} - t, \text{etc.}$ We can now interpret $[r_k, z_k, T_k]$ as the states, and $[R_k, y_k]$ as the measurements. Also,
note that before the deposition time for lot $k$ is computed, we also know $T_k$. Hence, what is not known on the onset of run $k$ are $\xi_k$, i.e., the actual deposition for lot $k-1$, and $r_k$, i.e., the deposition rate for lot $k$. The deposition time is fixed to belong to the set $t_k \in [-4.8, 20]$, where this restriction could be obtained via scheduling constraints. Furthermore, we assume that $r_k \in \mathcal{R}$, where $\mathcal{R}$ denotes the operating range of the equipment. Here, it is assumed that $\mathcal{R} = [-125, 300]$. The operating range denotes the range of parameter values over which the equipment is supposed to be operating. If the rate exceeds this range, it is assumed that a maintenance call will be placed. The controller can raise a maintenance alarm by checking the information state, since during normal operation we should have $p_k(r, \xi, T) = -\infty$ for all $r \notin \mathcal{R}$. Hence, if $p_k(r, \xi, T) \neq -\infty$ for some $r \notin \mathcal{R}$, a maintenance alarm is issued. Note that from an implementation point of view $-\infty$ represents a large negative number. For this simulation, we have chosen $-\infty$ to be $-1 \times 10^{10}$.

Now let $\mu_k$ (the variation in the mean deposition rate) be zero mean, Gaussian, with standard deviations $\sigma_k = 2$. The instantaneous rate variation $\mu_k$ is modeled by taking the sum of two random variables. One from a uniform distribution over $[-12, 12]$, and the other from a zero mean Gaussian distribution with a standard deviation of one. The Gaussian component is used to model outliers. Also, the measurement error is modeled as being uniformly distributed between $-10$ and $10$ Å. Such a distribution models the resolution limit of the measurement apparatus. Placing bounds on the various noise sources ($v_k \in [-3 \sigma, 3 \sigma], w_k \in [-15, 15], \text{ and } n_k \in [-10, 10]$), we obtain the following set-valued dynamical system

\[
\begin{align*}
\tau_{k+1} &\in \tau_k \cap [-6, 6] \\
\xi_{k+1} &\in \tau_k \xi_k + \bar{\tau}_k + \bar{\tau}_k + (\bar{t} + t_k) [-15, 15] \\
R_{k+1} &\in \mathcal{T} \\
y_{k+1} &\in \tau_k y_k + \bar{\tau}_k + \bar{y}_k + (\bar{t} + t_k) [-3, 3] + [-10, 10].
\end{align*}
\]

Furthermore, we define the regulated output $z_k$ as

\[z_{k+1} = 0.1 \left( \xi_{k+1} - R_{k+1} \right)^2\]

where we assume that there is no cost associated with the control action. Here 0.1 is simply a scaling factor. Note that since we have no information on the power spectrum of the noise, we try to attenuate the influence of the noise on the regulated output over all frequencies. However, the above framework can take care of this case when the noise has a known power spectrum. In that case, we augment the above system with filters to emphasize those frequencies where the noise is active. See [11] for such an example.

Now, assuming that $\tau_k \in \mathcal{R}$, we solve the state feedback problem (3) assuming that $V(r, \xi, T) = 0$ for all $r \notin \mathcal{R}$. We iteratively test different values of $\gamma$, and the smallest value of $\gamma$ which yields $V(0, 0, 0) = 0$ is found to be approximately 20.2. Hence, this is the guaranteed value which is true for the nominal deposition over the entire operating range $\mathcal{R}$. It is clear that the value of $\gamma$ depends on the operating range $\mathcal{R}$. In fact, the smaller the range $\mathcal{R}$ the smaller the value of $\gamma$. The values of $V$. and the corresponding control policy $u_k$ are stored in a table. Note, that this computation is carried out offline. We assume that the initial rate is within $\pm 20$ Å/min of the nominal value $\bar{r}$, and hence initialize the information state as $p_0(r, \xi, T) = 0$ for $r \in [-20, 20]$, or $p_0(r, \xi, T) = -\infty$ otherwise. The certainty equivalence controller (6) is then implemented. We also consider the case having a measurement delay, where the measurement is now given by

\[
y_{k+1} \in \tau_k \gamma r_k - \gamma \tau_k + \tau_k + \bar{\tau}_k + \bar{\gamma} + (\bar{t} + t_k) [-15, 15] + [-10, 10]
\]

where $\tau$ is the measurement delay in lots. For this case we implement the controller obtained via (7).

We first consider the case where there is no delay. For purposes of illustrating some of the properties of the robust controller, we will employ a simple controller obtained via the EWMA estimate of the deposition rate. For this EWMA based controller, we will assume that the initial deposition rate is known exactly. This controller is implemented as

\[a_k = \lambda \left( \frac{\bar{y}_k}{\bar{t}_{k-1}} \right) + (1 - \lambda) a_{k-1}\]

\[i_k = \frac{\bar{t}_k}{a_k}\]

with $a_0$ equal to the exact initial value.

Fig. 2 illustrates the behavior of the controlled and uncontrollable deposition thicknesses, when the equipment is subject to random drift till run 10. After run 10, the statistics for $v_k$ are changed to a Gaussian distribution with mean 3 and standard deviation 1. The target is fixed at 1500 Å. Fig. 2 (top) shows the uncontrollable trajectory (corresponding to deposition for 5 min), and the trajectory generated by the robust controller. Since the initial rate is not known exactly for the robust controller, the controller undergoes an initial transient. The plot also illustrates that the noise induced by the robust controller is small. However, the controller still tracks the target accurately. This point is made clearer if one compares the trajectory generated by the robust controller, to those generated by the EWMA controllers for $\lambda = 0.2$, and $\lambda = 1.0$. These trajectories are shown in Fig. 2 (bottom). The mean and
Fig. 3. End-pointing: process subject to exceptional shifts.

Table 1
END-POINTING: ERROR STATISTICS OF THE CONTROLLERS

<table>
<thead>
<tr>
<th>Controller</th>
<th>$\lambda$</th>
<th>Mean</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>EWMA</td>
<td>0.2</td>
<td>35.13</td>
<td>52.73</td>
</tr>
<tr>
<td>EWMA</td>
<td>1.0</td>
<td>11.43</td>
<td>72.74</td>
</tr>
<tr>
<td>Robust</td>
<td></td>
<td>5.45</td>
<td>53.70</td>
</tr>
</tbody>
</table>

standard deviation (STD) of the errors incurred employing the different controllers is given in Table I. The error is defined as:

\[
\text{error} = \text{actual deposition} - \text{target}
\]

One immediately observes that the mean of the error incurred by the robust controller is comparable to that obtained by the high $\lambda$ EWMA controller, whereas the standard deviation of the error is comparable to that obtained via a low $\lambda$ EWMA controller. This point is also observed in Fig. 3, which shows the response of the robust and the EWMA ($\lambda = 1.0$) controllers to process shifts. The uncontrolled trajectory (deposition time 5 min) is also plotted. Note that from run 21 onwards, the perturbations of the deposition thickness around the target are magnified (compare the robust control trajectory to the uncontrolled trajectory). This is explained by the fact that since the rate for this period is below the nominal rate, the deposition time has to be increased to meet the target thickness. This amplifies the perturbations induced by the rate variations from wafer to wafer.

We now turn to the case with measurement delays. Fig. 4 shows the controller response to multiple shifts, with and without measurement delay. As one expects, the performance degrades with increasing delay. The main point we would like to make is that if the delay is too large (as compared to the interval between consecutive shifts), one may have to relax the performance requirements imposed on the system. For example, a delay of five lots in the current scenario would yield 12 lots of scrap, since the controller is not aware of the shift until six lots have been processed. Finally, Fig. 5 displays the case when one needs to carry out target tracking. We again observe that there is increased variation in the deposited thickness for higher deposition time (corresponding to a target thickness of 2000 Å).

B. LPCVD Rate Control

In this section, we will briefly consider the inverse problem, i.e., of controlling the rates in an LPCVD reactor. The model we work with is an experimentally determined one presented in [19]. Here, we limit our attention to the deposition on the first and last wafer. We augment the models with drift terms. The models express the deposition rates in terms of deposition temperature $T$, deposition pressure $P$, and the silane flow rate $Q$. They are given by

\[
\dot{\hat{R}}_1 = \exp(c_1 + c_2 \ln P + c_3 T^{-1} + c_4 Q^{-1}) + d_1
\]

\[
\dot{\hat{R}}_2 = \frac{1 - S' C_{gs} \hat{R}_1 Q^{-1}}{1 + S' C_{gs} \hat{R}_1 Q^{-1}} + d_2
\]

with the rates expressed in Å/min, $P$ in mtorr, $T$ in K, and $Q$ in sccm. The parameters are given [19] to be $c_1 = 20.65$, $c_2 = 0.29$, $c_3 = -15189.21$, $c_4 = -47.97$, $S' = 4777.8$. [20]
and $C_{gs} = 1.85 \times 10^{-5}$, where we have dropped the units for convenience. $d_1$, and $d_2$ represent the drift terms. The actual rates ($R_1$ and $R_2$) are obtained from the above model by adding a zero mean noise to $\hat{R}_1$ and $\hat{R}_2$. The noise is assumed to be Gaussian with a variance of 9. Furthermore, we assume that the maximum drift expected between runs is 0.3. This actually represents a shift of $\sigma$ in ten runs, and may be too large to be true in practice. However, we choose this value since it enables us to see the corrective action of the RbR controller in a fewer number of runs. The targets $T_1$ for $R_1$, and $T_2$ for $R_2$ are fixed at 169.75 $\text{A/min}$ and 141.7 $\text{A/min}$, respectively. It is also assumed that the target of the model do not undergo changes from run to run. We also assume that what is to be controlled are the measured rates ($R_1$ and $R_2$), since we do not have any information of the actual rates. We can express the system in a set-valued form as done in the previous example. The exact equations are

$$
c_1 [k + 1] - c_1 [k]$$

$$
c_2 [k + 1] = c_2 [k]$$

$$
c_3 [k + 1] = c_3 [k]$$

$$
c_4 [k + 1] = c_4 [k]$$

$$
S' C_{gs} [k + 1] = S' C_{gs} [k]$$

$$
d_1 [k + 1] \in d_1 [k] + [-0.3, 0.3]$$

$$
d_2 [k + 1] \in d_2 [k] + [-0.3, 0.3]$$

$$
R_1 [k + 1] \in f (P[k], T[k], Q[k]) + [-9, 9]$$

$$
R_2 [k + 1] \in f (P[k], T[k], Q[k]) g(Q[k]) + [-9, 9]$$

$$
y_1 [k + 1] \in f (P[k - \tau], T[k - \tau], Q[k - \tau]) + [-9, 9]$$

$$
y_2 [k + 1] \in f (P[k - \tau], T[k - \tau], Q[k - \tau]) g(Q[k - \tau]) + [-9, 9]$$

where

$$f(P[k], T[k], Q[k]) = \exp (c_1 [k] + c_2 [k] \ln P[k] + c_3 [k] T[k]^{-1} + c_4 [k] Q[k]^{-1})$$

and

$$g(Q[k]) = \frac{1 - S' C_{gs} [k] f(P[k], T[k], Q[k]) Q[k]^{-1}}{1 + S' C_{gs} [k] f(P[k], T[k], Q[k]) Q[k]^{-1}}$$

where we have not shown the dependence of $f$ and $g$ on the parameters ($c_1$, $c_2$, etc.) for convenience. Here $c_1$, $c_2$, $c_3$, $c_4$, $S' C_{gs}$, $d_1$, $d_2$, $R_1$, $R_2$ represent the states, and $y_1$, $y_2$ represent the measurements. Also $\tau \geq 0$ represents the measurement delay. We assume that the operating region of the equipment $R$ is $\pm 20\%$ around the parameters (i.e., $c_1$, $c_2$, etc.) and with the drifts $d_1$, and $d_2$ restricted to $[-30, 30]$. The regulated output is given by

$$e[k + 1] = (R_1[k + 1] - T_1)^2 + (R_2[k + 1] - T_2)^2.$$  

Solving the state feedback problem (after centering the system around the origin), and forcing the control inputs (i.e., $P$, $T$, and $Q$) to lie in the experimental design space [19], we obtain the value of $\gamma$ as 30. We then implement the certainty equivalence controllers given by (6) for the case of no delay, and (7) for the case of one lot delay.

For the purposes of simulation, we allow the drift to be zero mean, Gaussian with a standard deviation of 0.1. We force a process shift during run 4. This shift corresponds to a change in $c_3$ of 0.5, $c_4$ of $-0.7$, $C_{gs}$ of $1 \times 10^{-5}$, $d_1$ of 2.5, and $d_2$ of 11. After this, we force bad data points during run 10, by forcing $\tau_1 = 20$, and $\tau_2 = -20$. Finally, we subject the process to a steady drift of 0.3 for both $R_1$ and $R_2$ between runs 15–30. Fig. 6 (top) illustrates the controller performance for $R_1$, and Fig. 6 (bottom) for $R_2$. The horizontal lines show the target, the solid line is the case when we have no measurement delay, and the dotted line is the case when we have one lot delay. It is observed that the controller effectively compensates against these disturbances. We observe that the shift is compensated for in the very next run in case of no delayed measurements, and after an additional run in the delayed measurement case.

We now consider the process under drift alone. The reason for doing so is that it allows us to illustrate the corrective actions of the controller. Fig. 7 shows the trajectories generated by
controller induced noise. Furthermore, the framework is capable of handling nonideal noise statistics (in particular uncertain statistics), as well as multiple lot delayed measurements. Simulation results based on end-pointing and rate control in an LPCVD reactor were presented. An actual implementation is desirable to further validate the methodology. A crucial issue is the separation of the disturbances into nominal (which we include in the model) and exceptional. Taking very large bounds on the disturbances will yield controllers, which rarely experience exceptional disturbances but are too conservative, and fail to meet the desired performance objectives. This, and the development of efficient computational algorithms is currently being looked into. We would like to point out that in [1], a robust controller was presented for polynomial models which is a conservative approximation to the controller presented here.

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REFERENCES

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